

The pricing record: one option, six inputs, one of them unobservable

Chapter 2. Before any model: where each input comes from, and who can check it. Five are facts or quotes; the sixth is an opinion, and it decides the price.

Desk		Prepared by	
Instrument		Date	

Input	Where it comes from	Kestrel 100	Your option	Source, and who checked it
Spot	Observable: the screen	100.00		
Strike	The contract	100.00		
Maturity, years	The calendar	1.00		
Risk-free rate	A market quote	4.00%		
Dividend yield	A forecast	0.00%		
Implied volatility	Nobody can observe it	24.00%		
Contracts	The ticket	10,000		

Derived at once	Formula	Kestrel 100	Your option
d1	$(\ln(S/K) + (r - q + \sigma^2/2) T) / (\sigma \sqrt{T})$	0.286667	
d2	$d1 - \sigma \sqrt{T}$	0.046667	
$N(d1), N(d2)$	the cumulative normal	0.612816, 0.518611	
Price	$S \exp(-qT) N(d1) - K \exp(-rT) N(d2)$	11.454068	
Value of the lot	price x contracts	114,540.68	

If the volatility line has no source and no second signature, the price has neither. Chapter 9 puts a number on it: eight points of volatility move this price by 3.061643, 26.7 per cent.

The closed form, built on one page

Chapter 2. Every intermediate on its own line, so the price reads as a construction. The three identities at the foot must hold before the price is used for anything.

Desk		Prepared by	
Instrument		Date	

Step	Kestrel 100	Your option	Checked
$\ln(S/K)$	0.000000		
$(r - q + \sigma^2/2) T$	0.068800		
$\sigma \sqrt{T}$	0.240000		
$d1$	0.286667		
$d2$	0.046667		
$N(d1)$	0.612816		
$N(d2)$	0.518611		
$S \exp(-qT) N(d1)$	61.281622		
$K \exp(-rT) N(d2)$	49.827554		
Call price	11.454068		

Identity that must hold	Kestrel 100	Your option	Holds?
Put by parity: $C - S \exp(-qT) + K \exp(-rT)$	7.533012		
Put by its own formula, less the put by parity	0.000000000		
Zero-volatility floor: $\max(S \exp(-qT) - K \exp(-rT), 0)$	3.921056		
Break-even daily move: $\sigma S / \sqrt{365}$	1.2562		

A price that sits below the floor, or a put by parity that differs from the put by formula in the ninth decimal, is a bug, not a market view.

The lattice convergence log

Chapter 4. Price the option at rising step counts and record the error against the closed form. The error oscillates while the nodes cross the strike, then halves with every doubling.

Desk		Prepared by	
Instrument		Date	

Steps	Price	Error	Error over error at half the steps	Your price	Your error
10	11.220175	-0.233893	-		
25	11.543355	0.089287	-		
50	11.406747	-0.047321	-0.53		
100	11.430376	-0.023691	0.50		
250	11.444584	-0.009484	-		
500	11.449325	-0.004743	0.50		
1,000	11.451696	-0.002372	0.50		

Reading the log	Kestrel 100	Yours
Steps past which the error ratio settles at 0.50	100	
Error at 1,000 steps	-0.002372	
Steps needed for an error below 0.001 (error x steps is constant)	2,372	

A lattice that does not halve its error when the steps double has a bug in its probabilities or its payoff, whatever its price looks like.

The simulation run report

Chapter 5. No simulated price leaves the desk without its standard error beside it. The band is the price plus or minus 1.96 standard errors; the last column says whether it contains the closed form.

Desk		Prepared by	
Instrument		Date	

Paths	Price	Payoff SD	Standard error	Difference	In SEs	95% band	Contains?
1,000	10.452260	16.702	0.528164	-1.001808	-1.90	9.4171 to 11.4875	yes
10,000	11.284141	17.286	0.172860	-0.169927	-0.98	10.9453 to 11.6229	yes
100,000	11.344755	17.388	0.054986	-0.109313	-1.99	11.2370 to 11.4525	no
1,000,000	11.424367	17.408	0.017408	-0.029701	-1.71	11.3902 to 11.4585	yes

Question to ask of any run	Kestrel 100	Your run
Seed, and were the runs independent?	Not stated; pooled, the four sit 2.36 SEs low	
Paths for a standard error of one cent	3,030,385	
Paths for an SE as small as the lattice's error at 1,000 steps	53,867,596	
Variance reduction used	none reported	

Precision grows with the square root of the paths: a tenfold run buys about three times the precision.

The model comparison memo

Chapter 7. Three models, one option. Separate what the models disagree about from what one run of one model has not yet converged on, then set both against what the input moves.

Desk		Prepared by	
Instrument		Date	

Model	Price	Less the closed form	Of which noise	Your figures
Closed form	11.454068	-	-	
Lattice, 1,000 steps	11.451696	-0.002372	none	
Simulation, 1,000,000 paths	11.424367	-0.029701	1.71 standard errors	

The comparison	Kestrel 100	Yours
Spread across the three, highest less lowest	0.029701	
as a share of the price	0.2593%	
Disagreement between the models, noise removed	0.002372	
Eight points of volatility, 20 to 28	3.061643	
Ratio, input over the spread as printed	103	
Ratio, input over the models' own disagreement	1,291	

The quantitative industry argues about models because a model can be argued about rigorously. The number that decides the price arrives in the same cell every morning and is never defended.

The Greeks report, with a second path to each

Chapter 8. Each sensitivity from its formula and again by finite difference from the pricing function itself. If the two columns disagree beyond the sixth decimal, one of them is wrong.

Desk		Prepared by	
Instrument		Date	

Sensitivity	Analytic	Finite difference	Step	Difference	Your analytic	Your FD
Delta	0.612816	0.612816	0.0001	5.8e-09		
Gamma	0.015953	0.015953	0.001	1.4e-08		
Vega, per point of volatility	0.382882	0.382882	0.0001	1.1e-09		
Theta, per day	-0.018048	-0.018058	1 day	9.5e-06		
Rho, per point of rate	0.498276	0.498276	0.0001	5.0e-09		

Identity	Kestrel 100	Yours
Vega = $\gamma \times S^2 \times \sigma \times T / 100$	0.382882	
Delta between nought and $\exp(-qT)$	0.612816	
Gamma not negative	0.015953	

In the workbook as first published, the finite-difference delta read -4,980.55: a missing bracket, found by exactly this comparison. Make the step far smaller and the column measures floating-point noise; far larger and it measures curvature.

The risk report line: the lot in currency

Chapters 8 and 15. The form in which a sensitivity reaches a risk report: per unit, on the lot, and the gamma-theta exchange that decides whether a long-gamma position earns its keep.

Desk		Prepared by	
Instrument		Date	

Line	Per option	On the lot of 10,000	Your position
Delta	0.612816	6,128.16	
Gamma	0.015953	159.53	
Vega, per point of volatility	0.382882	3,828.82	
Theta, per day	-0.018048	-180.48	
Rho, per point of rate	0.498276	4,982.76	

Gamma and theta	Kestrel 100	Yours
Theta per day, raw	-0.018048	
Break-even daily move against raw theta	1.5042	
Theta per day, net of financing the hedge	-0.012588	
Break-even daily move against net theta	1.2562	
Daily move the volatility implies, $\sigma S / \sqrt{365}$	1.2562	

Net of financing, the break-even move equals the move the volatility implies: the option is fairly priced exactly when the market moves as much as the volatility says.

The volatility note

Chapters 9 and 16. Where the volatility came from, what a point of it is worth on this contract, and the implied volatility solved from a quote rather than typed in.

Desk		Prepared by	
Instrument		Date	

Item	Kestrel 100	Yours
Source of the volatility, and date	24 per cent, the desk's mark	
Value of one point, per option	0.382882	
Eight points, 20 to 28, per option	3.061643	
as a share of the price	26.7%	
Strike at which one-year vega peaks	107.12	

Volatility	Price	Vega at that volatility	Your price
16%	8.405688	0.377807	
20%	9.925054	0.381391	
22%	10.688823	0.382314	
24%	11.454068	0.382883	
26%	12.220179	0.383191	
28%	12.986697	0.383298	
32%	14.519595	0.383064	

Implied volatility from a quote	Kestrel 100	Yours
Quoted price	12.500000	
Pass 1 (from 24 per cent)	26.731732%	
Pass 2	26.730180%	
Price reproduced	12.500000	

An implied volatility is not a measurement. It is the number that makes a model reproduce a price, so quoting it is quoting the price in different units.

The library acceptance tests

Chapter 17. An afternoon of tests to run on a pricing library you did not write. Each has an expected answer that does not depend on trusting the library.

Desk		Prepared by	
Instrument		Date	

Test	Expected	Your library	PASS or FAIL
Put-call parity: $C - P - S \exp(-qT) + K \exp(-rT)$	0		
The put, by parity	7.533012		
Zero volatility gives the discounted intrinsic value	3.921056		
Deeply in the money: spot at 1000	903.921056		
Deeply out of the money: spot at 10	0		
Price rises with volatility	1		
Price rises with maturity	1		
Price falls as the strike rises	1		
Finite-difference delta agrees with the analytic delta to 1e-6	0.612816		
Vega equals $\gamma \times S^2 \times \sigma \times T / 100$	0.382882		
Implied volatility of the library's own price returns the input volatility	24.000000%		
Lattice error halves when the steps double (past 100 steps)	0.50		
Price at zero maturity is the intrinsic value	0.000000		

Most implementation bugs fail at least one of these. Library: _____ Version: _____ Tested by: _____

The hedge log, period by period

Chapter 12. Sell the option, hold delta in shares, finance the rest, and record every rebalance. The portfolio column is the hedging error to date.

Desk		Prepared by	
Instrument		Date	

Step	Time left	Spot	Delta	Shares traded	Cash	Portfolio
0	1.0000	100.00	0.6128	0.6128	-49.8276	0.0000
1	0.9808	104.19	0.6760	0.0632	-56.4474	-0.0436
2	0.9615	107.71	0.7246	0.0487	-61.7313	-0.0424
3	0.9423	110.49	0.7600	0.0354	-65.6872	-0.0074
4	0.9231	112.46	0.7837	0.0236	-68.3964	0.0506
5	0.9038	113.58	0.7969	0.0132	-69.9494	0.1224
6	0.8846	113.82	0.8004	0.0035	-70.4026	0.2002
7	0.8654	113.17	0.7942	-0.0062	-69.7542	0.2772

Step	Time left	Spot	Delta	Shares traded	Cash	Portfolio

Kestrel 100 on the workbook's path: 52 weekly rebalances, realised volatility 20 per cent, drift 6 per cent. At expiry the path ends at 107.97 and the hedge makes 0.7621 an option, 7,620.70 on the lot.

What the trade paid, and why

Chapter 13. Attribute the result of a delta-hedged option to the volatility gap, then compare it with the dispersion that remains even when the forecast is right.

Desk		Prepared by	
Instrument		Date	

Line	Kestrel 100	Your trade
Implied volatility sold	24.00%	
Realised volatility over the life	20.00%	
Gap, in points	4	
Vega per point	0.382882	
Expected profit, vega x gap	1.5315	
Simulated mean over many paths	1.5981	
Dispersion at the hedging frequency (monthly)	2.3031	
Edge over dispersion	0.69	
Result actually booked		
of which the volatility gap		
of which hedging residue		

Realised volatility	Analytic profit	Simulated mean	Your result
20%	1.5315	1.5981	
24%	0.0000	0.0004	
28%	-1.5315	-1.6022	

Change the direction of the stock and the result barely moves; change the realised volatility by two points and it moves a great deal. At a four-point edge hedged monthly, one year in four still loses (case two).

The rebalancing policy

Chapter 14. The frequency is a choice with a price on both sides. Record the residue at each frequency, the desk's cost of an adjustment and its price of residue, and the count they imply.

Desk		Prepared by	
Instrument		Date	

Rebalances a year	Residue, one option	Share of premium	Fall from the row above	Rule predicts	Your residue
12	2.3031	20.1%	-	-	
52	1.1512	10.1%	2.0006	2.0817	
252	0.5265	4.6%	2.1865	2.2014	
1,008	0.2632	2.3%	2.0004	2.0000	

The choice	Case three (ADDED inputs)	Your desk
Constant: residue x sqrt(count)	8.3579	
Cost of one rebalance on the lot	15.00 (ADDED)	
Charge a year per unit of standard deviation	0.20 (ADDED)	
Optimal count: $(\text{charge} \times \text{lot} \times \text{constant} / (2 \times \text{cost}))^{2/3}$	67.71	
Total cost at the optimum	3,047	
Total at weekly, above the optimum	1.7%	
Total at daily, above the optimum	58.6%	

Halve the cost of an adjustment and the optimal count rises by 1.5874; the total is flat around its minimum, so the price of residue matters more than the argument over daily against weekly.

The desk report line, and the questions to ask of it

Chapters 14, 15 and 17. The residue belongs beside the premium in the same breath. Then the questions that separate a price someone has checked from a price someone has typed.

Desk		Prepared by	
Instrument		Date	

Report line	Kestrel 100	Your book
Premium received on the lot	114,540.68	
Hedging frequency	daily, 252 a year	
One standard deviation of hedging residue	5,265.00	
hedged monthly instead	23,031.00	
Vega on the lot, per point	3,828.82	
Model spread, noise removed	0.002372	

Question	At Kestrel	On this book
Where does the volatility come from, and who signed it?	The desk's mark, 24 per cent	
What is a point of it worth on the position?	3,828.82 on the lot	
Which tests has the pricing library passed?	13 of 13 (document 9)	
What is the standard error of any simulated figure?	0.017408 at 1,000,000 paths	
How often is the hedge adjusted, and why that often?	Daily; optimum about 68 (case three)	
What does the residue do in an ordinary year?	5,265.00, one standard deviation	

Sold 114,540.68 of calls, hedged daily, one standard deviation of hedging residue 5,265.00. The second figure is not a stress scenario; it is what the position does in the ordinary course of events with every forecast correct.